Boats Technical Mini-Report
The design and manufacture of
"Bröther may I have some böats"
Written by Dieter Brehm

Brehm, Dieter   Daitzman, Sam

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1 Introduction

We present an explanation and example presentation of boat analysis and design. We constructed a boat purely using the results of this analysis.

2 Proposed Design

2.1 Shape Determination

We chose to define our boat hull shape by intersecting an ellipsoid with two planes as shown in Figure. 2. The base shape of an ellipsoid allows for fluid movement through the water, and after testing several shapes including the base parabola extrusion, the ellipsoid allowed for faster iteration of design and produced desirable RM-curve data. The first intersecting plane forms the top "deck" of the boat, which is essentially splitting the ellipsoid slightly off of half along the vertical y-axis. We added the second intersecting plane in order to increase stability of the boat at lower angles of x-axis tilt (called the heel angle of the boat system) which will help the boat float level when minimally disturbed. Mathematically, the boat region is defined by the bounds:

$$R_{Boat} = \left[\frac{x^2}{11.25} + \frac{(y - 10.24)^2}{11.25} + \frac{z^2}{20} < 1 \land 0 < y < 11.24, \{x, y, z\} \right], \tag{1}$$

Which when rendered in 3D looks like Figure. 2.

2.2 Ballast

We added 950 grams of ballast to our boat, located such that its center of mass is 1 cm above the bottom of the boat. We chose this ballast value based on analysis of the mass of hardboard our boat used in its construction and as a parameter of our righting moment curves.

2.3 Parameter Enumeration

In order to refine our boat design, we varied the ballast mass, location, and the ellipsoid fundamental radii length in the x, y, and z axes. The ballast is a weighted object which serves as a tool to shift the center of mass of the boat in any direction we desire depending on its position in the boat. Changing the shape of the ellipsoid via its parameters changes the total volume of the surface, thus

potentially changing its center of mass. Additionally, changing the shape of the object changes the shape and center of buoyancy of an underwater region when the boat is floating.

2.4   Boat Mass

One of the results of varying these parameters is a change of the total mass of the boat. We found the mass of the boat, isolated from the ballast and the mast, by integrating over the region of the boat with an estimated density. We estimated the density value by choosing a reasonable value, constructing a boat draft in SolidWorks, and then using the CAD model to find the mass of the used hardboard. With the mass of the material and the volume of the boat region, we found an average density over the region. With the total mass of the boat found, we then added the mass of the mast and of the ballast to our system. The total mass of the boat system is 1180 grams.

2.5   Boat Center of Mass

The center of mass is located at the vector 0.00016cm,6.29948cm,-0.00041cm. We found this value using the process described by the equations 3 through 7. The center of mass of the ballast and of the mast change this value, shifting it down or up relative to the centroid of the boat hull surface. In the system of this boat, the y axis is vertical. As shown in the vector position of the center of mass, there is a small error in the calculation of the center of mass of the boat hull. This is due to a design trade off we made during our programming. In order to increase simulation performance and iterate designs faster, we used a command in Mathematica which takes a symbolic representation of a region and turns it into a discrete region of many polygons. Integrating over such a region is much faster, but introduces slight error.

2.6   Visualization

Prior to the construction of the boat, we created a CAD (Computer Aided Design) model of the boat such that we could use the model to laser cut parts and assemble them. A render of this model is shown in Figure.1. The render includes flat hardboard rib parts and representations of the mast and the ballast of the boat.

Figure 1: Render created in SolidWorks of the boat assembly. Includes models for the hardboard rib construction, the mast, and a place-holder for the ballast.
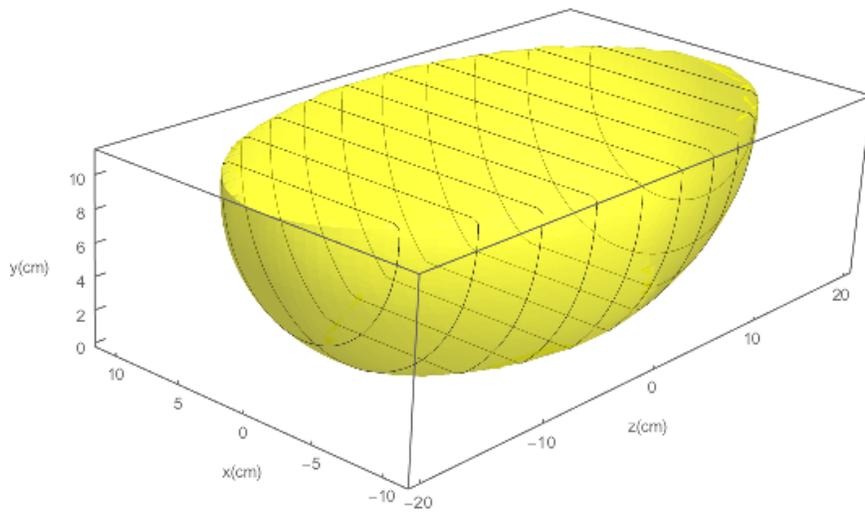

Figure 2: Three dimensional plot of the implicit region representing a boat. The origin is placed in the bottom middle of the boat and all units are in cm.

3 Design Justification

3.1 Boat Shape

Now, we will describe the process used to analyze the behavior of our boat shape when it is tipped to a given heel angle. In order to perform an analysis of boat characteristics, we must first clarify the region and an origin. Let the boat be defined by a number of bounds:

$$R_{Boat} = \left[\frac{x^2}{11.25} + \frac{(y - 10.24)^2}{11.25} + \frac{z^2}{20} < 1 \wedge 0 < y < 11.24, \{x, y, z\} \right],$$
(2)

which, when plotted in Mathematica, looks like Figure. 2. The origin is at the middle bottom of the boat.

3.2 Boat Mass

The goal of the following analysis is to predict the forces at play when a boat defined by equation. 2 is rotated at a variety of heel angles. In order to perform force and torque calculations regarding the boat region which will dictate the movement of the boat, we will need the mass of the region. We can find the mass of the region by performing the integration:

$$\iiint_{R_{Boat}} \rho_{Boat} \, dR_{Boat},$$
(3)

where R_{Boat} is the region of the boat defined in three dimensions and ρ_{Boat} is the density of the boat.

3.3 Boat Center of Mass

Next, we must find the center of mass of the boat in order to find the point about which the righting moment will act. In order to find the center of mass, we perform the integrations:

$$\text{COM}_{\text{XBoat}} = \frac{1}{M_{Boat}} \times \iiint_{R_{Boat}} x \, dR_{Boat},$$
(4)

$$\text{COM}_{\text{YBoat}} = \frac{1}{M_{Boat}} \times \iiint_{R_{Boat}} y \, dR_{Boat},$$
(5)

$$\text{COM}_{\text{ZBoat}} = \frac{1}{M_{Boat}} \times \iiint_{R_{Boat}} z \, dR_{Boat},$$
(6)

which gives the components of a vector which represents the COM of the boat. The result is a vector

$$\vec{COM} = \begin{bmatrix} COM_{\text{XBoat}} & COM_{\text{YBoat}} & COM_{\text{ZBoat}} \end{bmatrix}.$$
(7)

3.4 Displaced volume

Calculating the quantity of how much water the boat displaces lets us perform a similar calculation to the COM ones in equations 4, 5, 6 for the center of buoyancy.

Using the buoyancy model, which states that the force of buoyancy is equal to the force of gravity when the average density of the object is less than that of water:

$$\mathbf{F_b} = -\mathbf{F_g}, \tag{8}$$

$$\mathbf{F_g} = M_{boat}g, \tag{9}$$

$$\rho_{\text{Water}} = 1000\text{kg/m}^3, \tag{10}$$

Where F_b is the force due to buoyancy, F_g is the force due to gravity, and ρ_{Water} is the density of water. If its density were more than that of water, the object would be in a sinking state. We can find the Displaced volume with

$$V_{\text{Displaced}} = \frac{F_g}{\rho_{Water} \times g}, \tag{11}$$

Which yields a volume $V_{Displaced}$.

3.5 Finding the Water Line

In this system, we represent the water line where the top of the water intersects the boat as a line of the form.

$$y = \tan\theta x + d, \tag{12}$$

where $\tan\theta$ is the slope of the line, and d is the draft, or y-intercept, of the line. This draft represents the distance from the water line to the bottom of the center of the boat. In order to find the center of buoyancy in the next section, we need the region that represents the intersection of the boat region and the water. We can find this region by solving for the equation of our boat hull in term of the draft d. In order to solve for d, we reference the equation for the volume displaced in the previous section (equation. 11.) Because the region defined by the waterline equation must be equal to the volume displacement found in the force calculation, we can set the equations equal and solve for variable d, the draft.

$$V_{\text{Displaced}} = \frac{F_g}{\rho_{Water} \times g} = \iiint\limits_{R_{Water}} dR_{Water}, \tag{13}$$

3.6 Boat Center of Buoyancy

We can find the center of buoyancy by integrating over the underwater region in a similar way to finding the center of mass. The components can be found by integrating over the underwater region in terms of draft d like so:

$$COB_{\text{YBoat}} = \frac{1}{V_{Displaced}} \times \iiint\limits_{R_{Water}} x \, dR_{Water}, \tag{14}$$

$$COB_{\text{XBoat}} = \frac{1}{V_{Displaced}} \times \iiint\limits_{R_{Water}} y \, dR_{Water}, \tag{15}$$

assuming that the center of buoyancy in the z axis is at 0 given that we are not considering pitch angles. In this case, it is important to note that the implicit region R_{Water} is defined with the variable d which we solved for in equation 13. The result is a vector

$$C\vec{O}B = \begin{bmatrix} COB_{\text{XBoat}} & COB_{\text{YBoat}} & 0 \end{bmatrix}. \tag{16}$$

which is the location in 3d space of the center of mass of the displaced water volume. We call this location the center of buoyancy.

3.7   Righting Moment

The righting moment of the system at heel angle θ would then be

$$||R\vec{M}|| = (C\vec{O}B - C\vec{O}M) \times \begin{bmatrix} -F_b \sin\theta & F_b \cos\theta & 0 \end{bmatrix}. \tag{17}$$

If run on several angle values, we can generate a plot which represents a Righting Moment vs. Angle curve such as Figure. 3. The angle of vanishing stability is when the curve crosses the angle axis. At this point the force switches from being a restoring (righting) force to being a destabilizing force. Different boat designs result in different AVS values, which means different levels of stability. Ballast can be strategically added to boats to make them more stable by lowering the center of buoyancy.

For this boat region, shown in Figure. 2, the angle of vanishing stability is approximately 124 degrees. We know that the boat will not flip immediately because the net moment at zero degrees is zero. Additionally,
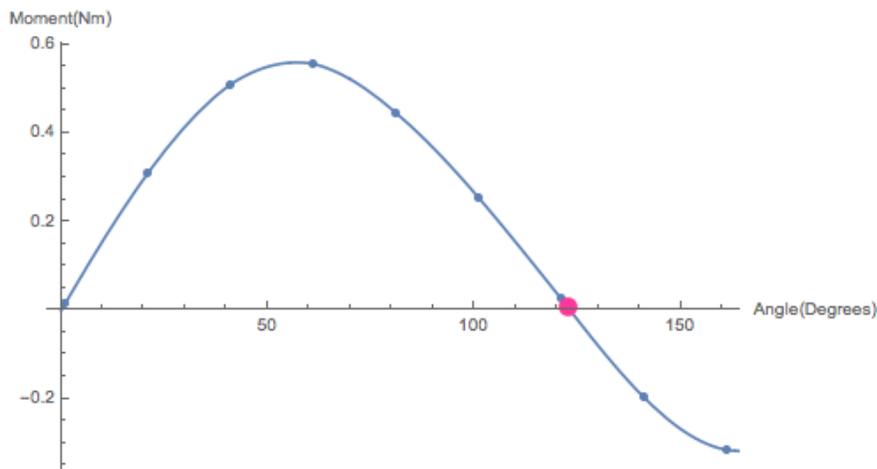

Figure 3: Plot of moment in Nm versus Heel Angle in degrees of an example boat. The Angle of Vanishing Stability is located at y=0 and is approximately 124 degrees. A fit line has been added to the equation and a red dot is used to identify the AVS point.

3.8   Heel Angle Variation

To summarize the actions of the center or buoyancy and mass, we generated renders of our simulation at heel angles of 0 degrees, 30 degrees, and 110 degrees. These simulation results are shown in Figure. 4. In addition to the positions of the centers of masses, the water and boat surfaces are included to clarify the visualization.
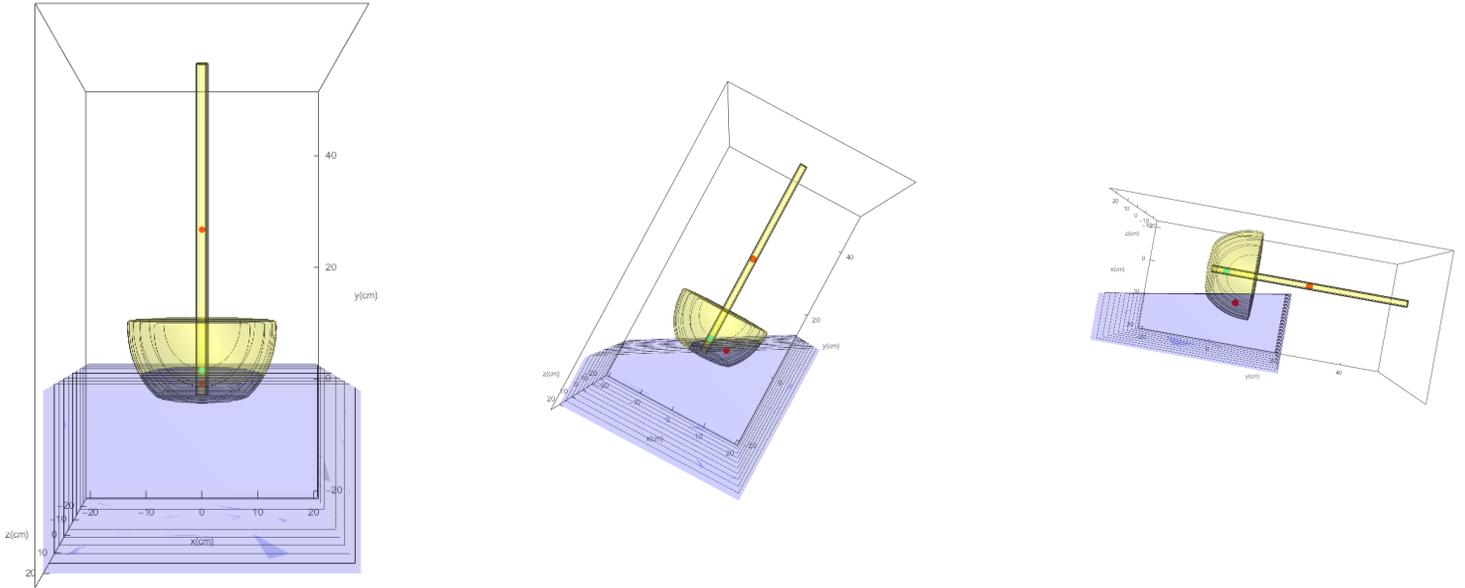
Figure 4: Visualization of the center of buoyancy (bottom red dot), center of mass(green dot), and center of mast mass (top red dot) at several heel angles. The heel angles used in the simulation are 0, 30, and 110 degrees respectively. Additionally included is a visualization of the water, mast, and boat surfaces. All units are in cm and the origin is in the bottom middle of the boat.

4   Expected Performance

Based on our simulated stability curve in Figure. 3, our boat will float flat (the mast perpendicular to the water). At an angle of zero degrees, the net torque is zero.

Our stability curve predicts an angle of vanishing stability of 124 degrees. This is lower than we originally intended, but time constraints limited further optimization. Regardless, this is within the acceptable range of the design requirements presented.

Compared to other boats built in QEA, we expect that it will have a middling speed. It will be faster than boats which are built from a two dimensional extrusion, but slower than boats which are designed to cut through the water specifically. In our case, the ellipsoid surface is neither fully optimized for speed nor a worst case.